Teaching proof in a dynamic geometry environment: what mediation?

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Abstract

This paper aims to discuss two issues. First we intend to analyse if and how dynamic geometry software prove to be a communication space for students and how they support students towards proving. Second, we reflect about the use of video-tapes as source of information both for Mathematics Education research and for the teaching practice. In particular we approach the question: how do video-tapes help in the construction of a shared experience in the classroom?

Resumé

Ce travail se propose de réfléchir sur deux thèmes. En premier lieu nous voulons analyser si et comment l'emploi d'un logiciel en géométrie dynamique peut se proposer comme un espace de communication pour les étudiants et comment cela peut les aider dans l'activité de démonstration. En deuxième lieu nous réfléchissons sur l'emploi des vidéo – enregistrements en qualité de source d'information soit dans la recherche, soit dans la pratique didactique. En particulier on considère la question suivante : dans quelle façon les vidéo – enregistrements peuvent nous aider dans la construction d'une expérience partagée en classe.

Introduction

"To be mathematically literate means to understand mathematics as a part of the cultural heritage. Mathematical experiences have to be offered to all students, that include knowledge about the cultural and the historical background of mathematics." (Manifesto CIEAEM, 2000). Mathematical *literacy* involves both the aspects of understanding and communicating. Moreover, it is important to consider the necessity of a literacy *for all* the students, so that everyone is given the possibility of participating into a mathematical experience. Nowadays, the presence of technology in our society influences the school and the educational change. The research that has been carried out over the years suggests that, within the problem of mathematical literacy for all, the main issues to be addressed, with respect to the use of new technologies, concern the potentialities they may offer, compared to more traditional tools, with respect to specific contexts of use. One of these potentialities may be to foster and widen the possibilities of understanding and communicating mathematical knowledge. The part of that knowledge we focus on in this paper concerns the theoretical organisation of mathematics, in its aspects of understanding and communicating. The chosen domain is Euclidean geometry and the technology we used is the microworld Cabri-Géomètre.

The "treasure hunting problem"

The research project we have been involved in for some years now within the Mathematics Education group at the University of Turin and the Graduate School of Education in Bristol, is aimed at investigating the potentialities of the dynamic geometry software Cabri-Géomètre (Baulac et al, 1988) in supporting students' proving process in geometry (e.g. Arzarello et al, 1998). By proving process we mean the process of exploring, conjecturing and proving in an open problem. The research methodology adopted involves the use of videotaping in the classroom, that provides a rich account of all actions and language of the students.

In this paper we present the analysis of the video of a classroom session which was part of a classroom experiment[1] that involved five Further Mathematics A-level students in Bristol. It was aimed at introducing students to the proving process in geometry within a dynamic geometry environment. The teacher run most sessions. The students have a solid mathematical background. They did geometry only at GCSE. They had never used Cabri before. In this experiment, we carried out a brief introduction of Cabri, but we did not cover all the possibilities it offers. The only thing we explained them was the dragging test. After that, other Cabri features were introduced during the sessions when needed, so to make them part of the classroom culture.

In the session we analyse in this paper, the students were asked to tackle the following problem (the problem was proposed and solved by George Gamow in: "Uno due tre…infinito" Arnoldo Mondadori Editore, 1952, Milano):

A man left instructions about how to discover the place where a treasure had been hidden in an island, knowing only the position of three trees: a pine, an oak and an apple tree. In order to discover the treasure a man has to position himself where the apple tree is and then:

1. *walk till the oak counting the steps, then turn right[2] (90 degrees), walk the same number of steps and put a mark where he arrives.*
2. *come back to the apple tree, walk to the pine counting the steps, then turn left (90 degrees), walk the same number of steps and put a mark where he arrives.*

The treasure will be in the midpoint of the line joining the two marks.
The man who had these instructions goes to the island; when he gets there he finds out that the apple tree is missing. Will he be able to discover the treasure?

In the session, the students are divided in pairs. They are asked to work with paper and pencil first until they get a conjecture, and then they can work in Cabri. The session lasts 1 hour and 20 minutes. Almost all the students get to a conjecture. In the following session (1 hour and 30 minutes) they are asked to go on working on this problem and try to construct a proof.

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[1] The classroom experiment was carried out by G.Moenne, C.Mogetta, F.Olivero and R.Sutherland from the Graduate School of Education, University of Bristol, in collaboration with Churchill School, Bristol.

[2] The position of the oak and the pine with respect to the apple tree was fixed, so that there was no ambiguity between turning right and left.

The process the observed[3] students followed in the solution of this problem can be divided into three phases. The first phase is carried out in paper and pencil. The students make the conjecture: "*M* (the treasure) *is independent from A"* (the apple tree) right at the beginning. In this first phase a logical conflict is present: does A (the apple tree) exist or not? The students make two different drawings, which represent two worlds: one with the apple tree and one without the apple tree. Then, they reconstruct the whole situation starting from two separate drawings, one containing the pine and one containing the oak. The second phase concerns the work in Cabri. The students start with a construction, which is not general, because the point corresponding to the apple tree is not constructed as a free point. After they realise this, they perform a new construction and carry out a lot of explorations, but they are not able to discover the procedure to construct M. This phase too is characterised by a logical conflict: all points can be moved, but what are the important relationships? The third phase concerns the attempts to proving that M is fixed. The students work with a static Cabri figure and they are not able to construct a proof.

In the following, we will analyse two short excerpts from the students' protocol, with respect to the following issues: does Cabri provide a communication space? Does it support the proving process?

Developing a communication space

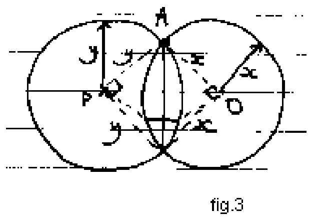
In this first excerpt, the students (I and S) are at the end of the paper and pencil phase, they are exploring the situation and try to make sense of what happens when the apple tree is not there.

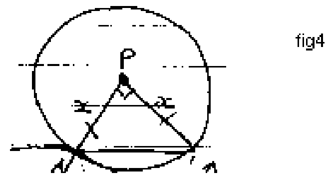
87. *I: a point. Then he walks there. So you have x and x. So that's..* I draws another figure (Fig.2)
88. *S: A.*
89. *I: if you put a circle around there …*
90. *S: <u>yeah, so what you are saying is</u> that here is A (*pointing on Fig1*), picking A as a point.*
91. *I: <u>no.</u>*
92. *S: no, picking that (*P on Fig1*) as the point.*
93. *I: so he walks out (*I points on fig1 from P to A*)…*
94. *S: to there.*
95. *I: so this is point P (*on Fig2*), this is A, you go from A to P, from P to there, the same distance.*
96. *S: <u>yeah, so if you make</u> that (*A in Fig1*) the centre, we have both (*pointing O and P*).*
97. *I: <u>no,</u> now you do the same for that point (*O on Fig1*).*

[3] We video-recorded one pair of students for each session.

98. S: yeah.

*99. I: right. Now he walks from O (*I draws Fig3 while saying*). You're gonna get exactly the same thing, with the circle around again. So that's point O, so that's A.*





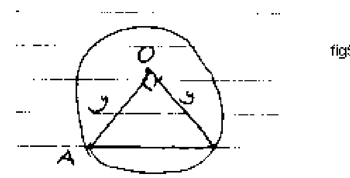


Figure 1 **Figure 2** **Figure 3**

In this paper and pencil phase, it is the student who has got the pencil (Ian) to guide the actions, while the other one (Stuart) shows difficulty in understanding and participating in what Ian wants to do. Ian does most of the talking, while Stuart tries to explain to Ian what he sees in the drawings, but he is always wrong: Ian always answers *No* (#91, #97).

In the second excerpt, which is in the second phase, the students are carrying out the first construction in Cabri.

205.S: you draw one circle as that… No you don't do that, you draw a point on that circle and then you draw a circle which goes through that point. Now check whether that point is linked, check whether when you change one the other changes.

206.I moves the first circle

207.S: yeah, so here we go. So this is what you want.

208.I put letters, segments PA and AO, then moves circle then P

209.S: Now you need a perpendicular line and you need a midpoint.

210.I constructs **Figure 4**.

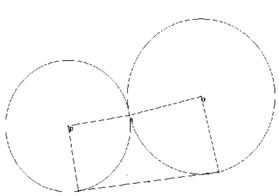


Figure 4

In Cabri, Ian has got the mouse, but Stuart tells him what to do. Now Stuart does most of the talking. It seems that in Cabri it is easier to enter one another universe of thought. It is not necessary to have the mouse. The one who has got the mouse moves a point, but both of them see the variation of the figure on the screen; this is something dynamic and so it allows interactive participation of both students.

Towards proving?

This extract is taken from the second phase, immediately after the students have finished the first Cabri construction.

211.S: So <u>now we have to prove</u> it stays exactly the same thing.??????????? Now <u>what do you do?</u>

212.As soon as he finished constructing I starts dragging the first circle to strange positions (**Figure 5**).

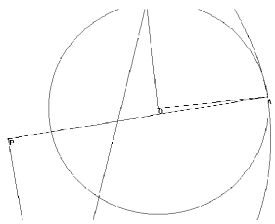


Figure 5

213.I: <u>wow!</u>

214.S: but <u>how do you prove</u> this?

215.I <u>keeps dragging</u> fast.

*216.S: it is still the same point (*laughing*)*

217.I: <u>wow!</u>

In this episode the students are observing what happens in Cabri at two different levels. Stuart wants to prove; he is at a theoretical level. He seems not to be surprised to see that the treasure stays fixed while moving the apple tree. On the contrary Ian is immersed in Cabri and keeps dragging and observing what happens to the treasure, being surprised (*wow!)*. Stuart seems also disturbed by this continuous dragging, and keeps saying that they have to prove it. He knows *that* the treasure is fixed, but he wants to know *why* this is so.

Conclusions

The comparison of the first two excerpts suggests that the Cabri environment proves to be a communication space for students, that is a space in which the interaction between the students is more likely to converge towards a shared understanding than in the paper and pencil environment. Cabri provides "a setting in which the emerging knowledge [of the students] can be expressed, changed and explored … and in which the language that A and B can now use to communicate is the language of the medium" (Noss & Hoyles, 1996). It seems that entering one another universe of thought in Cabri is easier than in paper and pencil. Seeing the dynamic variation of figures on the screen allows interactive participation of both students to the same experience. This dynamic variation is extremely stimulating from the perceptive point of view, but students can interpret this in different ways. In the third excerpt mentioned above, Ian remains at a perceptual level, as he keeps dragging the figure and looking at the variation on the screen, while Stuart immediately thinks of how to prove what he sees on the screen. We believe that Cabri is a good support, because it allows students to do explorations and experiences, which foster the production of conjectures and motivate to proving: proving *why* a certain proposition holds, within a theory, after they know *that* it is true, within the Cabri environment (see the second example). Moreover, the fact that when constructing a proof the students exploit their previous exploration in Cabri suggests that this tool may support the construction of a cognitive unity (Mariotti et al, 1997) between conjectures and proofs.

Developing a Cabri shared experience may constitute a common reference point when constructing the proof. And, generally speaking, constructing a shared classroom experience may be very productive and meaningful. A means to develop this common experience, through the communication in the classroom, can be found in the use of videotapes. In fact, "by videotaping a session and watching the video with the students, we might reduce the risk that significant moments of the classroom life are neglected or forgotten by students over time. The teacher himself may hear voices would remain hidden in a normal classroom and may exploit these recovered voices to discuss problematic issues" (Furinghetti et al, 2001). From a research point of view, the use of video-tapes allows to carry out deep observation of dynamics and processes in the classroom. The implication of the analysis are both at cognitive level, through the development of frameworks about cognitive processes, and at practical level, through suggestions for classroom practice.

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